

6강 소성역학

6강 목차

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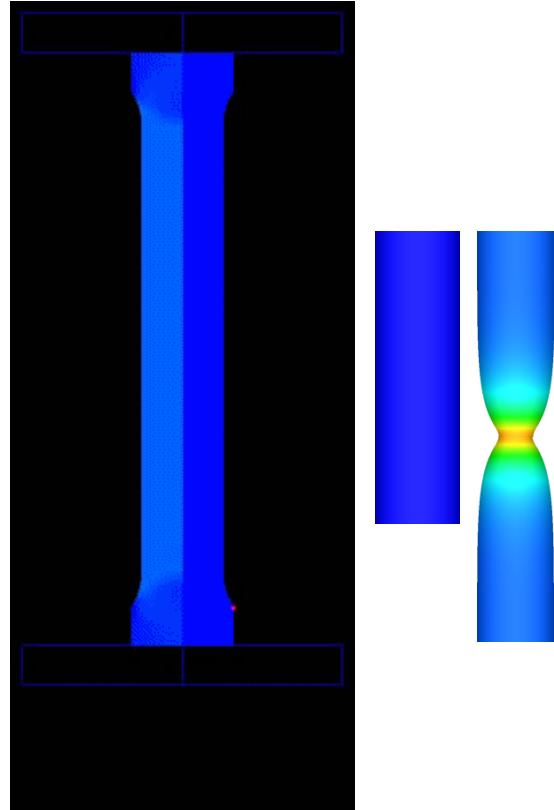
6.5 열전도방정식 및 마찰, 역학 총정리

1.

인장시험과 역학기초

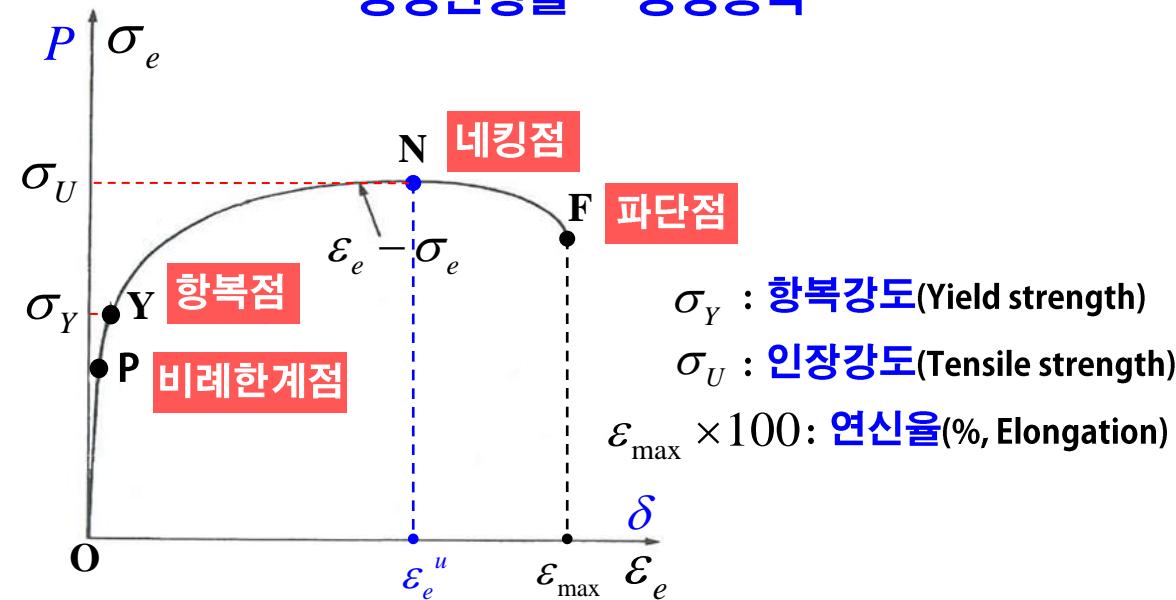
인장시험

ε_e 와 σ_e 의 정의



$$\varepsilon_e = \frac{\delta}{L_0}, \quad \sigma_e = \frac{P}{A_0} \quad \cdot e: \text{engineering}$$

공칭변형률 공칭응력



인장시험의 해석

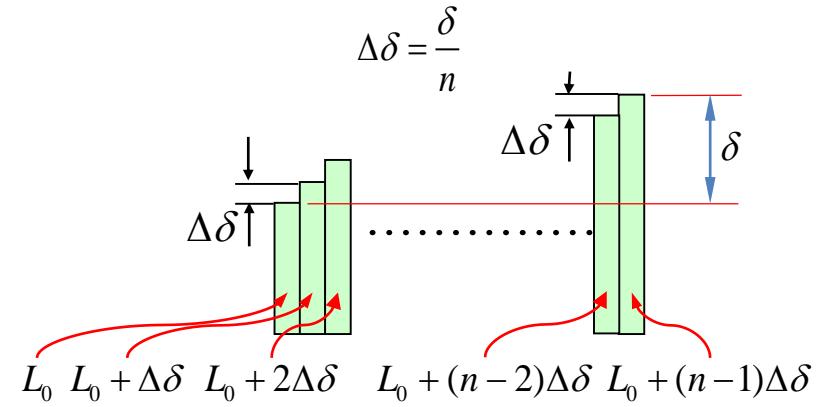
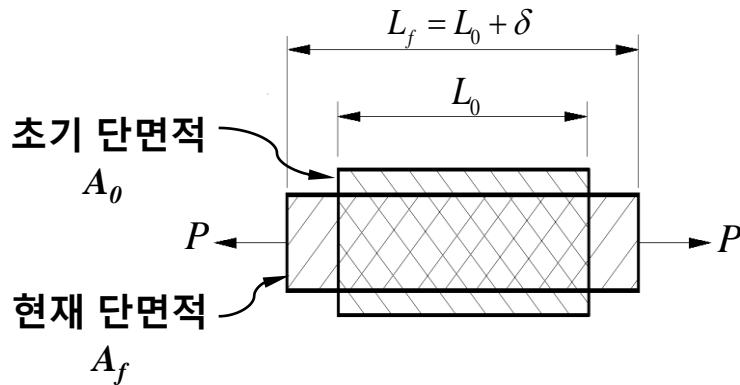
공칭응력-공칭변형률곡선

▣ 진응력

$$\sigma_e = \frac{P}{A_0} \leftarrow \text{공칭응력}$$

$$\sigma_t = \frac{P}{A_f} = \frac{P}{A_0} \cdot \frac{A_0}{A_f} = \sigma_e \cdot \frac{L_f}{L_0} = \sigma_e (1 + \varepsilon_e) \leftarrow \text{진응력}$$

$A_0 L_0 = A_f L_f$



$$\begin{aligned}\varepsilon_t &= \frac{\Delta\delta}{L_0} + \frac{\Delta\delta}{L_0 + \Delta\delta} + \dots + \frac{\Delta\delta}{L_0 + (n-1)\Delta\delta} \\ &= \sum_{i=1}^n \frac{\Delta\delta}{L_0 + \Delta\delta(i-1)} = \int_{L_0}^{L_f} \frac{dL}{L} \\ &= \ln \frac{L_f}{L_0} = \ln \left(\frac{L_0 + \delta}{L_0} \right) = \ln \left(1 + \frac{\delta}{L_0} \right) \\ &= \ln(1 + \varepsilon_e)\end{aligned}$$

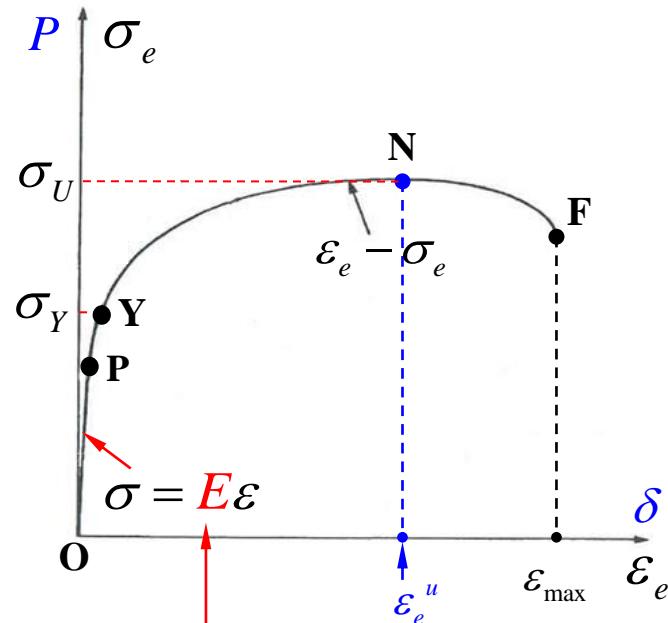
▣ 진변형률

$$\varepsilon_e = \frac{\delta}{L_0} \leftarrow \text{공칭변형률}$$

$$\varepsilon_t = \ln \frac{L_f}{L_0} = \ln(1 + \varepsilon_e) \leftarrow \text{진변형률}$$

▣ 탄성(OY 사이)

$$\varepsilon_e = \varepsilon_t \equiv \varepsilon, \quad \sigma_e = \sigma_t \equiv \sigma$$



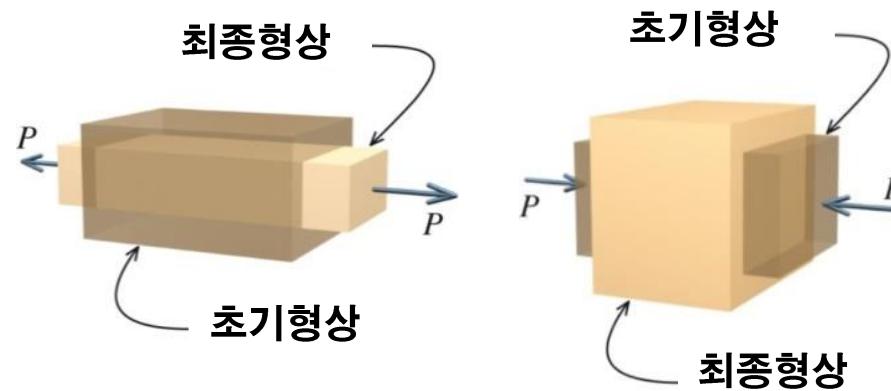
탄성계수(Modulus of elasticity)영
률(Young' s modulus)

▣ 단축하중에서 Hooke 법칙(OP 사이)

$$\square \sigma_t = E\varepsilon_t, \quad \sigma = E\varepsilon$$

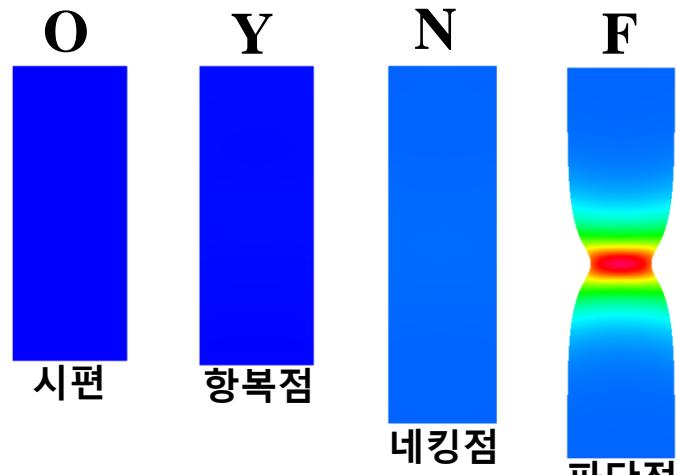
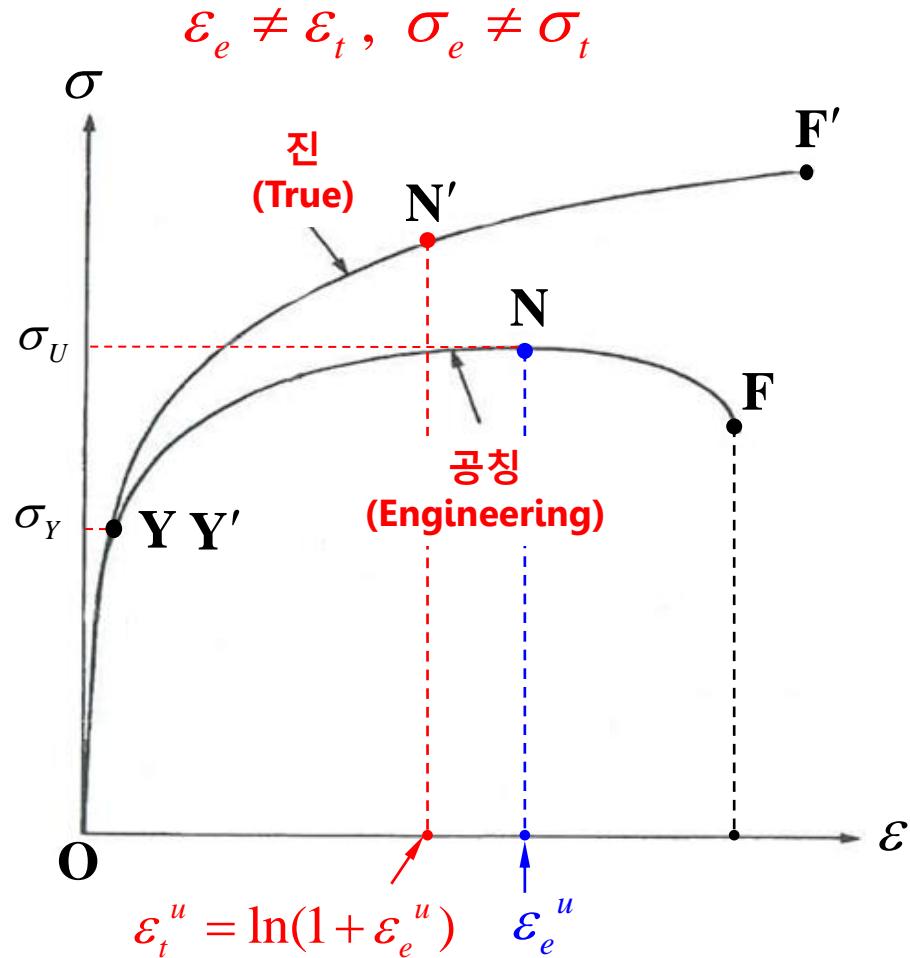
$$\square \sigma_x = E\varepsilon_x, \quad \sigma_{xx} = E\varepsilon_{xx}$$

▣ 포아송비(Poisson's ratio)



$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{\varepsilon_t}{\varepsilon_a}$$

▣ 소성(YF 사이)

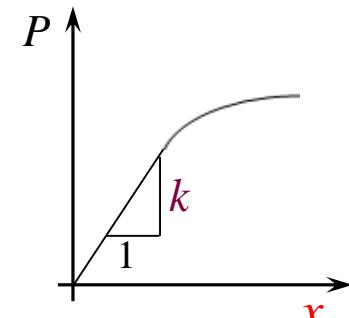


$$\square \varepsilon_e = \frac{\delta}{L_0}, \quad \sigma_e = \frac{P}{A_0}$$

$$\square \varepsilon_t = \ln(1 + \varepsilon_e)$$

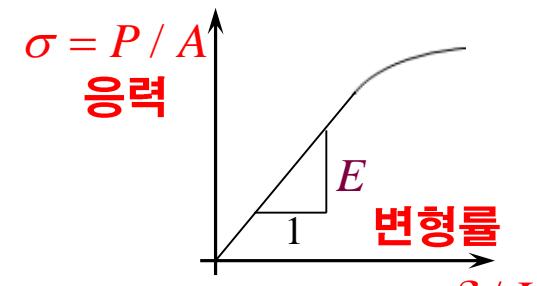
$$\square \sigma_t = \sigma_e(1 + \varepsilon_e)$$

스프링



k 스프링상수

봉, 트러스 부재



E 탄성계수, 영률

힘-변형의 관계

Hooke 법칙

$$P = kx$$

$$k_{eq} = \frac{AE}{L}$$

$$\sigma = E\varepsilon, \frac{P}{A} = E \frac{\delta}{L}, P = \frac{AE}{L} \delta$$

$$x = \frac{1}{k} P$$

변위

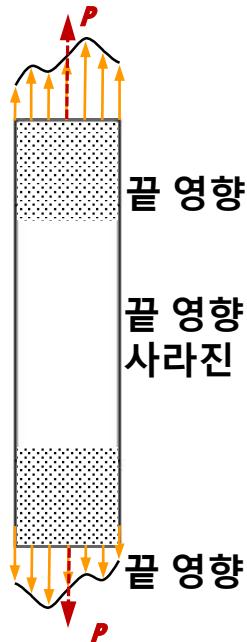
$$\delta = \frac{L}{AE} P$$

$$U = \frac{1}{2} kx^2 = \frac{1}{2} \frac{1}{k} P^2$$

변형에너지 (Strain energy)

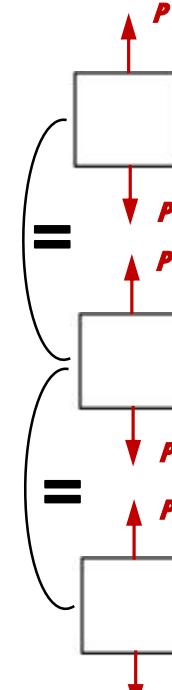
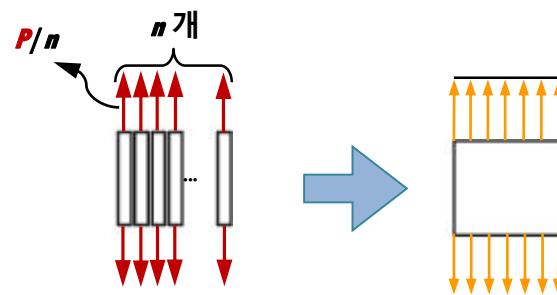
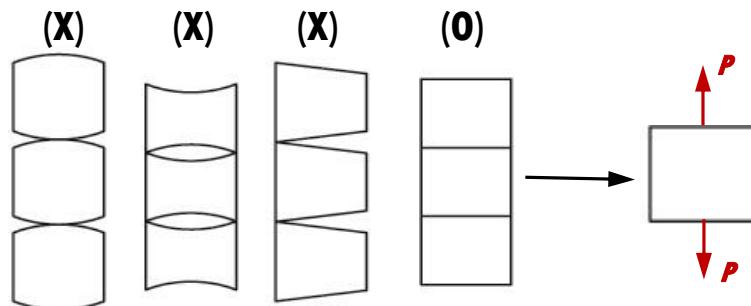
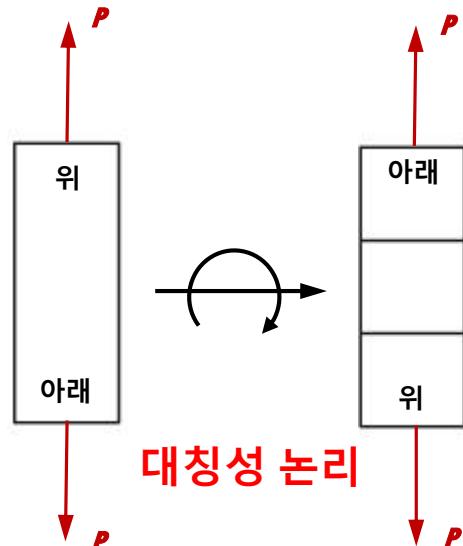
$$U = \frac{1}{2} k_{eq} \delta^2 = \frac{1}{2} \frac{L}{AE} P^2$$

▣ 네킹 직전까지 인장시편의 내력 – 단축하중 문제

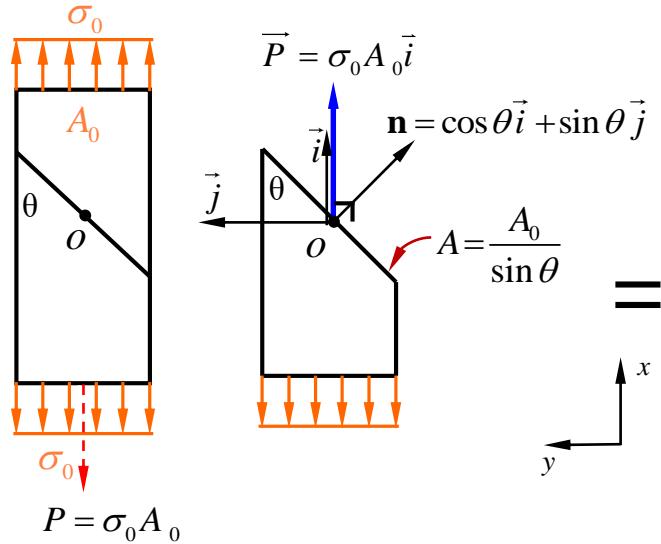


Saint-Venant 원리

두 개의 정역학적으로 동일한 힘 계는 힘이 작용하는 영역에서 멀어지면서 두 힘 계의 영향의 차이가 급속도로 사라짐



인장시험에서 내력과 응력벡터

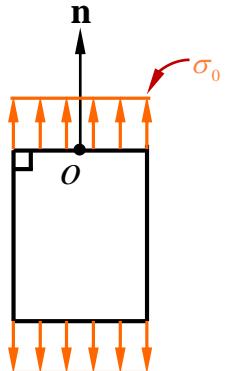


응력벡터, Stress vector

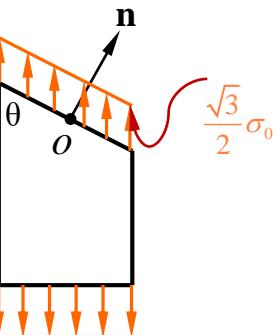
$$\overset{(n)}{\mathbf{T}} = \mathbf{t}^{(n)} = \frac{\vec{P}}{A} = \sigma_0 \sin \theta \vec{i}$$

$$\begin{aligned} & \sigma_0 \sin \theta \\ & \sigma_0 \cos \theta \sin \theta \square \sigma_{nt} \\ & \sigma_0 \sin^2 \theta \square \sigma_{nn} \\ \text{---} \\ \mathbf{t}^{(n)} = \sigma_0 \sin^2 \theta \vec{n} + \sigma_0 \cos \theta \sin \theta \vec{t} \end{aligned}$$

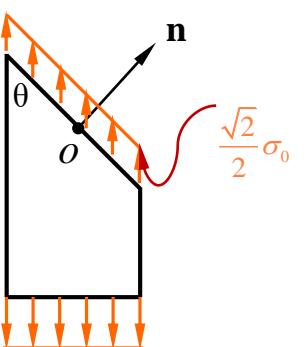
$$\theta = 90^\circ$$



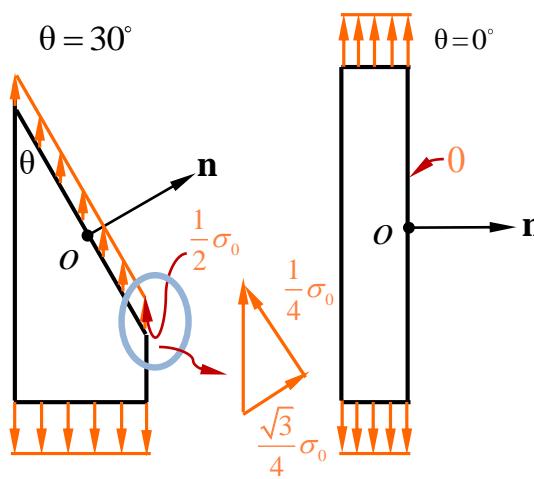
$$\theta = 60^\circ$$



$$\theta = 45^\circ$$

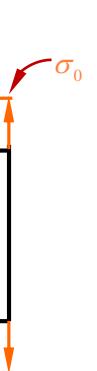


$$\theta = 30^\circ$$

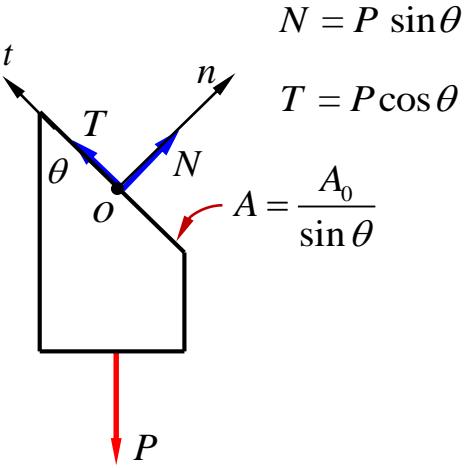
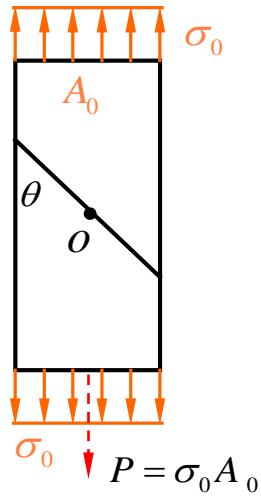


$$\theta = 0^\circ$$

$$\theta = 0^\circ$$



내력과 응력 성분



$$N = P \sin \theta$$

$$T = P \cos \theta$$

$$A = \frac{A_0}{\sin \theta}$$

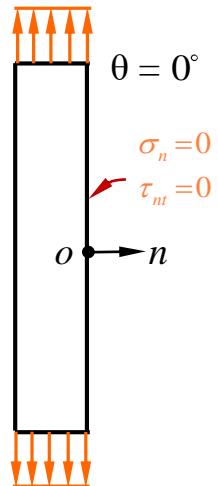
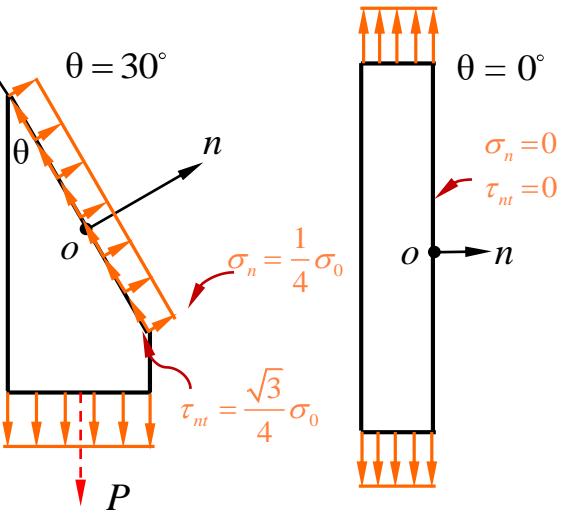
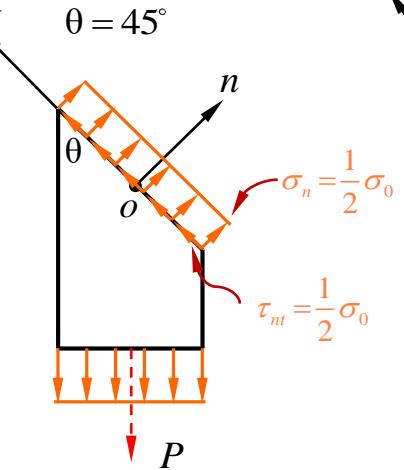
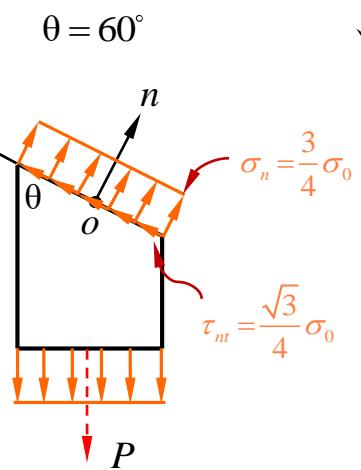
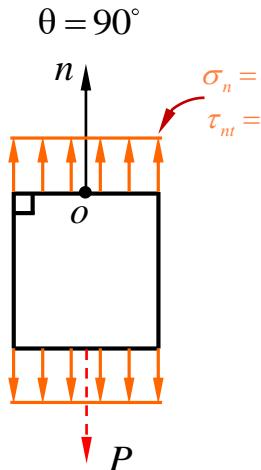
**법선응력
(Normal stress)**

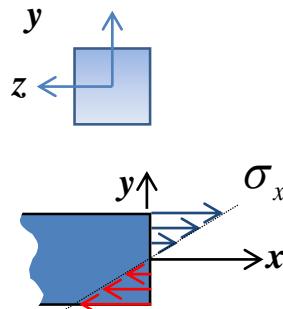
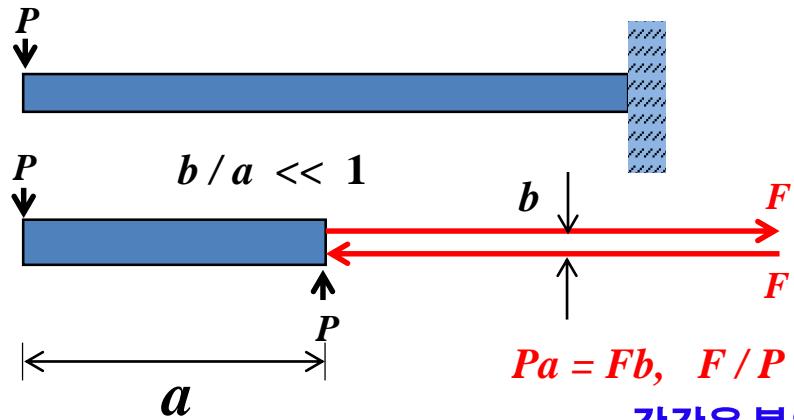
$$\sigma_{nn} = \frac{N}{A} = \frac{P}{A_0} \sin^2 \theta = \sigma_0 \sin^2 \theta$$

**전단응력
(Shear stress)**

$$\tau_{nt} = \frac{T}{A} = \frac{P}{A_0} \cos \theta \sin \theta = \sigma_0 \cos \theta \sin \theta$$

$$\mathbf{t}^{(n)} = \sigma_0 \sin^2 \theta \vec{n} + \sigma_0 \cos \theta \sin \theta \vec{t} = \sigma_{nn} \vec{n} + \sigma_{nt} \vec{t}$$





$\sigma_y = \sigma_z = 0$ 으로 가정

$\sigma_x = cy$ 로 가정

$$\sigma_x = E\epsilon_x$$

보의 인장/압축과 동일

$$\int_A y dA = 0 \quad \leftarrow \sum F_x = 0$$

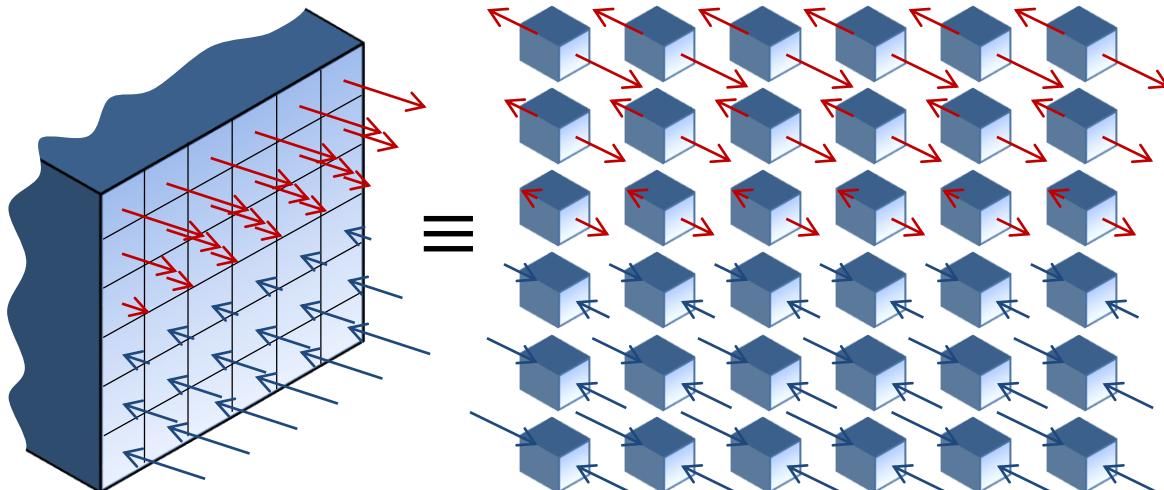
$$c = -\frac{M_b}{I_{zz}} \quad \leftarrow \sum M_z = 0$$

$$\sigma_x = -\frac{M_b y}{I_{zz}}$$

$$\epsilon_x = -\frac{y}{\rho} = -y v''(x)$$

$$EI_{zz} v''(x) = M_b(x)$$

$$I_{zz} \equiv \int_A y^2 dA$$

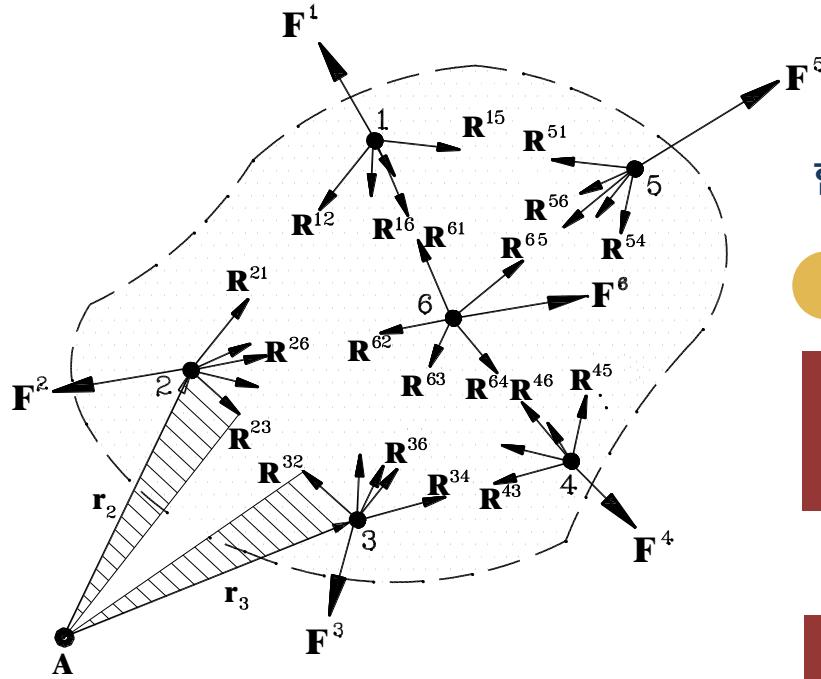


$Pa = Fb, \quad F / P = a / b, \quad F >> P$
각각은 봉의 인장압축 문제!

2. 이온력



뉴톤의 운동법칙



· \mathbf{F}^i : 질점 i 에 작용하는 모든 외력의 합

· \mathbf{R}^{ij} : 질점 i 가 질점 j 에 가하는 내력

평형조건식

$$\sum \mathbf{F}^i = \mathbf{0}$$

$$\sum_i \mathbf{r}_i \times \mathbf{F}^i = \mathbf{0}$$

$$\sum \mathbf{F} = \mathbf{0}$$

$$\sum \mathbf{M}_A = \mathbf{0}$$

“ 실제 물체의 질점 수는 무한대! ”

힘은 내력(질점 간의 힘)과 외력(하중, 반력, 자중 등)으로 구성

Newton의 운동법칙

제2 법칙

하나의 질점에 작용하는 모든 힘의 합, 즉 \mathbf{f} 는 그 질점의 질량 m 과
가속도 \mathbf{a} 의 곱과 같다. 즉, $\mathbf{f} = m\mathbf{a}$.

$$\mathbf{F}^i + \mathbf{R}^{i1} + \mathbf{R}^{i2} + \mathbf{R}^{i3} + \mathbf{R}^{i4} + \mathbf{R}^{i5} + \mathbf{R}^{i6} = \mathbf{0}$$

제3 법칙

작용과 반작용의 법칙. 두 질점 간에 작용하는 두 내력은 동일한
크기와 작용선을 가지며, 방향은 반대이다.

$$\mathbf{R}^{ij} = -\mathbf{R}^{ji}$$

$$\mathbf{r}_2 \times \mathbf{R}^{23} = -\mathbf{r}_3 \times \mathbf{R}^{32}$$

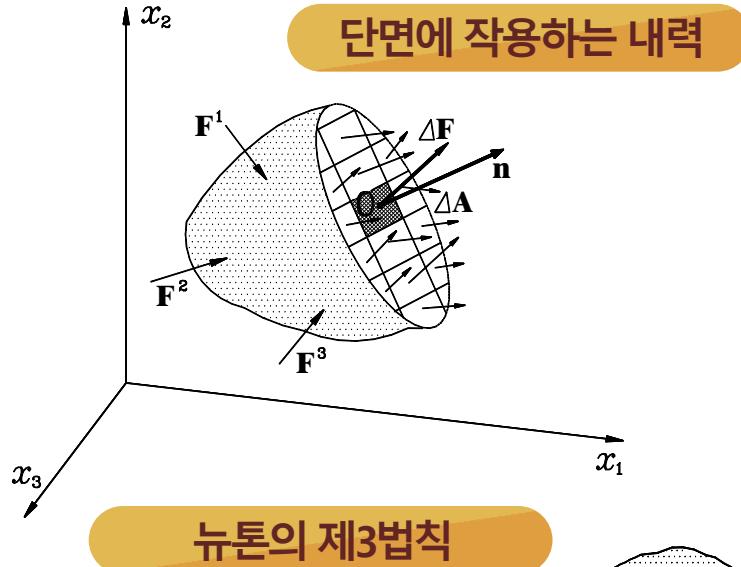
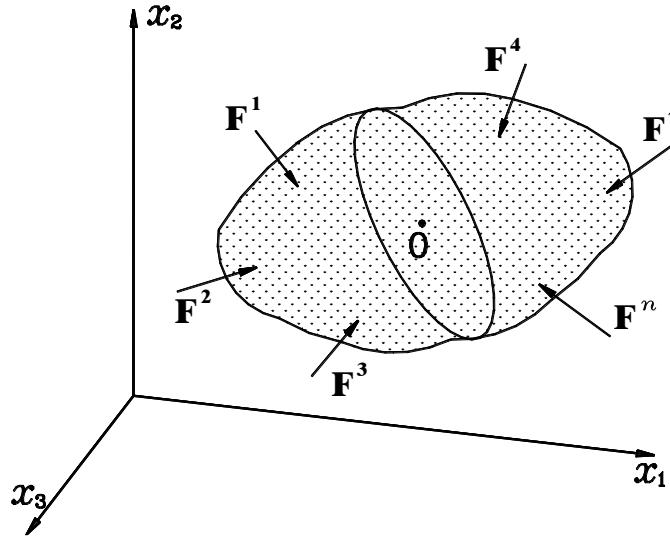
무한대 개의 방정식

2개의 백터 방정식(6개의 대수방정식)

이 평형조건식은 전체계는 물론이고 임의의 부분계에 대하여 성립해야 함.

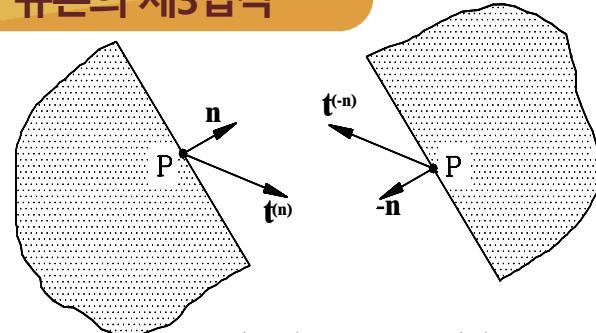
= 미소 선분, 면적, 체적 이것은 미분방정식으로 연결됨.

■ 응력벡터



$$\mathbf{T}^{(n)} = \mathbf{t}^{(n)} = \lim_{\Delta A \rightarrow 0} \frac{\Delta \mathbf{F}}{\Delta A}$$

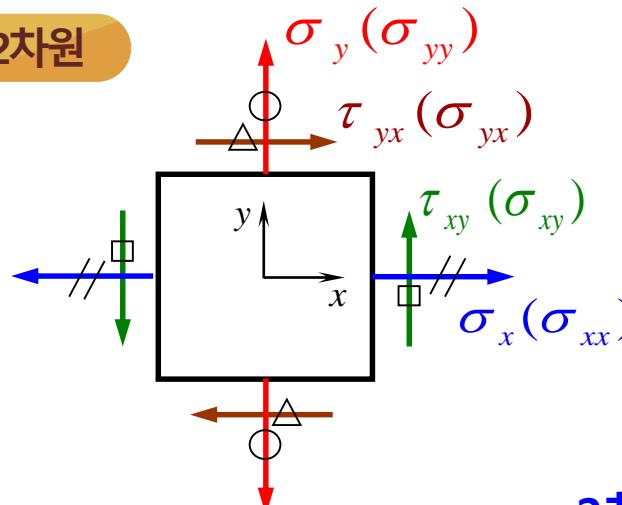
$$T_i^{(n)} = t_i^{(n)} = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_i}{\Delta A}$$



$$\begin{aligned}\mathbf{t}^{(-n)} &= -\mathbf{t}^{(n)} \\ t_i^{(-n)} &= -t_i^{(n)}\end{aligned}$$

응력성분

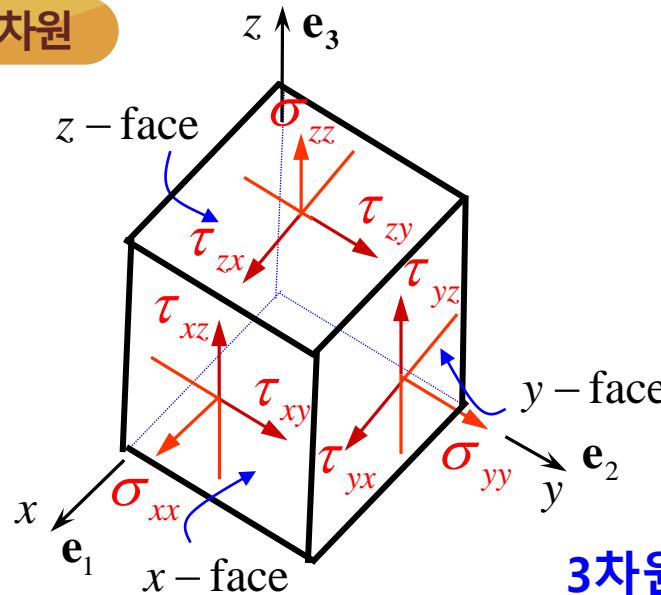
2차원



$$\mathbf{t}^{(i)} = \sigma_{xx}\mathbf{i} + \tau_{xy}\mathbf{j} \quad \text{평면응력(Plane stress)}$$

$$\mathbf{t}^{(j)} = \tau_{yx}\mathbf{i} + \sigma_{yy}\mathbf{j} \quad \begin{pmatrix} \sigma_{xx}(\sigma_x) & \sigma_{xy}(\tau_{xy}) & 0 \\ \sigma_{yx}(\tau_{yx}) & \sigma_{yy}(\sigma_y) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3차원

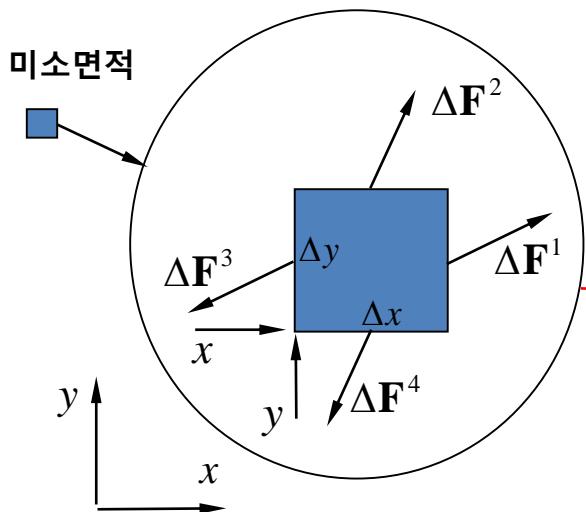


$$\mathbf{t}^{(e_i)} = \sigma_{i1}\mathbf{e}_1 + \sigma_{i2}\mathbf{e}_2 + \sigma_{i3}\mathbf{e}_3$$

$$t_j^{(e_i)} = \sigma_{ij} \quad \sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

3차원 역학에서의 점 = 정육면체

평형방정식



$$\sigma_{yy}(x, y + \Delta y) = \sigma_{yx}(x, y) + \frac{\partial \sigma_{yx}}{\partial y} \Delta y$$

$$\sigma_{xx}(x + \Delta x, y) = \sigma_{xx}(x, y) + \frac{\partial \sigma_{xx}}{\partial x} \Delta x$$

2차원(Plane stress, Plane strain, Axis-symmetric)

$$\sum F_x = 0 ;$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} = 0$$

$$\sum F_y = 0 ;$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\sum M_A = 0 ; \quad \sigma_{xy} = \sigma_{yx} \text{ 자중 무시}$$

3차원

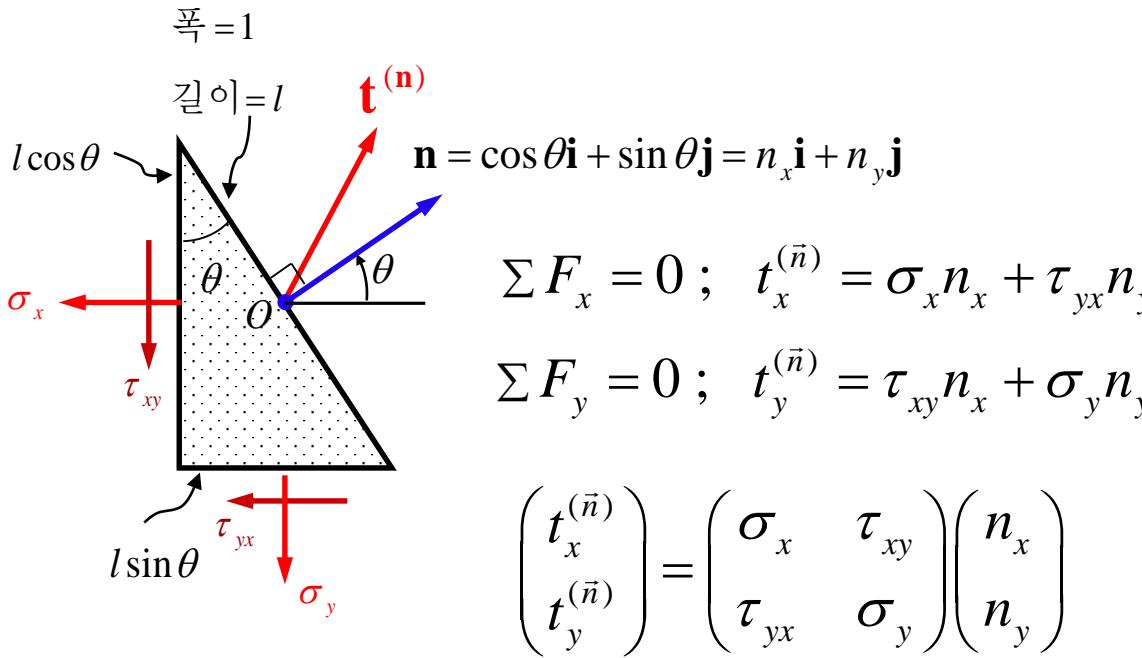
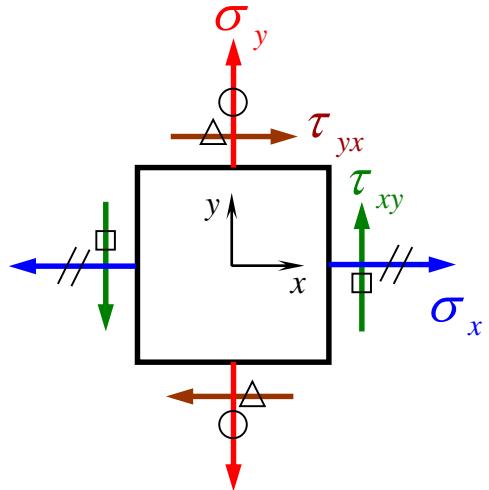
$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + f_x = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + f_y = 0$$

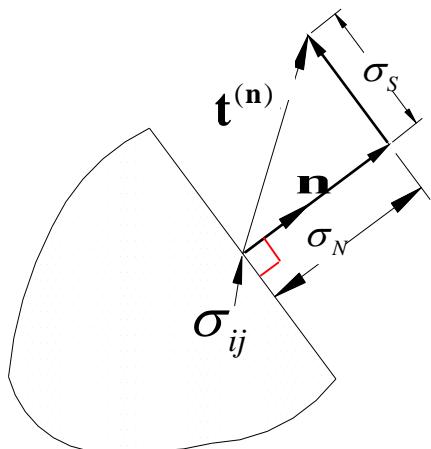
$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + f_z = 0$$

$$\sigma_{xy} = \sigma_{yx}, \quad \sigma_{yz} = \sigma_{zy}, \quad \sigma_{zx} = \sigma_{xz}$$

Cauchy 공식



$$\begin{pmatrix} t_x^{(\vec{n})} \\ t_y^{(\vec{n})} \end{pmatrix} = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \begin{pmatrix} n_x \\ n_y \end{pmatrix}$$



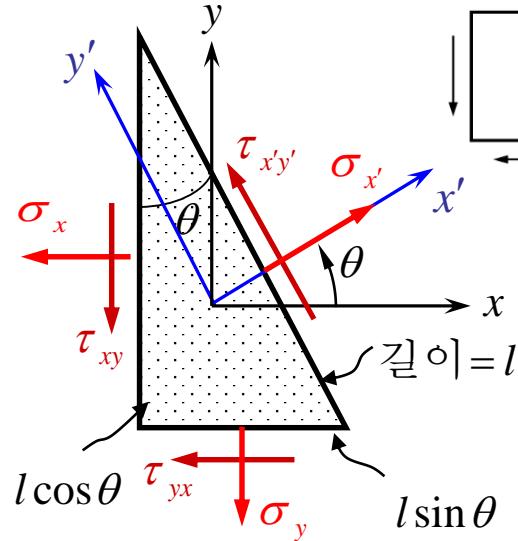
$$\begin{bmatrix} t_x^{(\mathbf{n})} \\ t_y^{(\mathbf{n})} \\ t_z^{(\mathbf{n})} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

Cauchy 공식

$$t_i^{(\vec{n})} = \sum_j \sigma_{ji} n_j$$

▣ 응력의 좌표변화

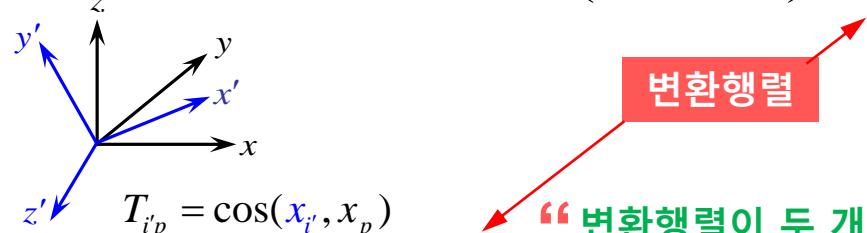
순수전단(Pure shear)



$$\sigma_{x'} = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cdot \cos \theta$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y) \cos \theta \cdot \sin \theta - \tau_{xy} (\cos^2 \theta - \sin^2 \theta)$$

$$\begin{pmatrix} \sigma_{x'} & \tau_{x'y'} \\ \tau_{y'x'} & \sigma_{y'} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$



변환행렬

$$[\sigma'] = [T][\sigma][T]^T = [T][\sigma][T]^{-1}$$

“변환행렬이 두 개 사용되었으므로 2차 텐서임”

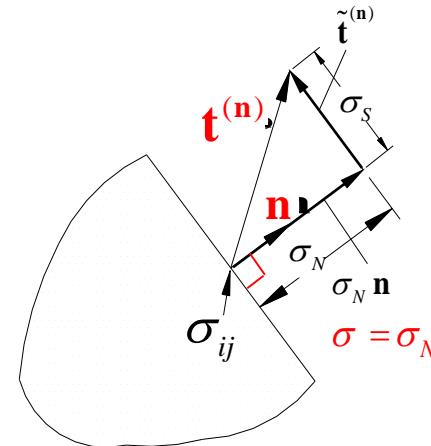
동질변환

$$\begin{pmatrix} \sigma_{x'x'} & \sigma_{x'y'} & \sigma_{x'z'} \\ \sigma_{y'x'} & \sigma_{y'y'} & \sigma_{y'z'} \\ \sigma_{z'x'} & \sigma_{z'y'} & \sigma_{z'z'} \end{pmatrix} = \begin{pmatrix} T_{x'x} & T_{x'y} & T_{x'z} \\ T_{y'x} & T_{y'y} & T_{y'z} \\ T_{z'x} & T_{z'y} & T_{z'z} \end{pmatrix} \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} T_{x'x} & T_{y'x} & T_{z'x} \\ T_{x'y} & T_{y'y} & T_{z'y} \\ T_{x'z} & T_{y'z} & T_{z'z} \end{pmatrix}$$

!!! $[\sigma']$ 와 $[\sigma]$ 의 고유치는 동일함 !!!

Cauchy의 공식

$$\begin{bmatrix} t_x^{(n)} \\ t_y^{(n)} \\ t_z^{(n)} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$



전단응력이 사라지는 조건

$$t^{(n)} = \sigma n$$

미지의 상수 :
법선응력, 방향 ($|n| = 1$)

주응력

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \sigma \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

고유치 문제

$$\begin{bmatrix} \sigma_{xx} - \sigma & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} - \sigma & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} - \sigma \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

■ 주응력과 응력불변치

고유치문제의 해법

특성방정식

$$\begin{vmatrix} \sigma_{xx} - \sigma & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} - \sigma & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} - \sigma \end{vmatrix} = -\left[\sigma^3 - I_1\sigma^2 - I_2\sigma - I_3 \right] = 0$$

응력불변치
(제1불변치, 제2불변치, 제3불변치)

주응력
 $\sigma = \sigma_1, \sigma_2, \sigma_3$

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = -\sigma_{xx}\sigma_{yy} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 = -\sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1$$

$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{xy}\sigma_{yz}\sigma_{zx} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2 = \sigma_1\sigma_2\sigma_3$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \sigma_i \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

주응력축
 $\Rightarrow \mathbf{n} = \mathbf{n}^{(i)}$

$\sigma' = \mathbf{T} \sigma \mathbf{T}^{-1}$
동질변환

$\mathbf{n}^{(i)} \perp \mathbf{n}^{(j)} = 0 \quad (i \neq j)$

유효응력

평균응력 σ_m 과 정수압 p

$$\sigma_m = \sum \sigma_{ii} / 3 = I_1 / 3 = -p$$

편차응력텐서 σ'_{ij}

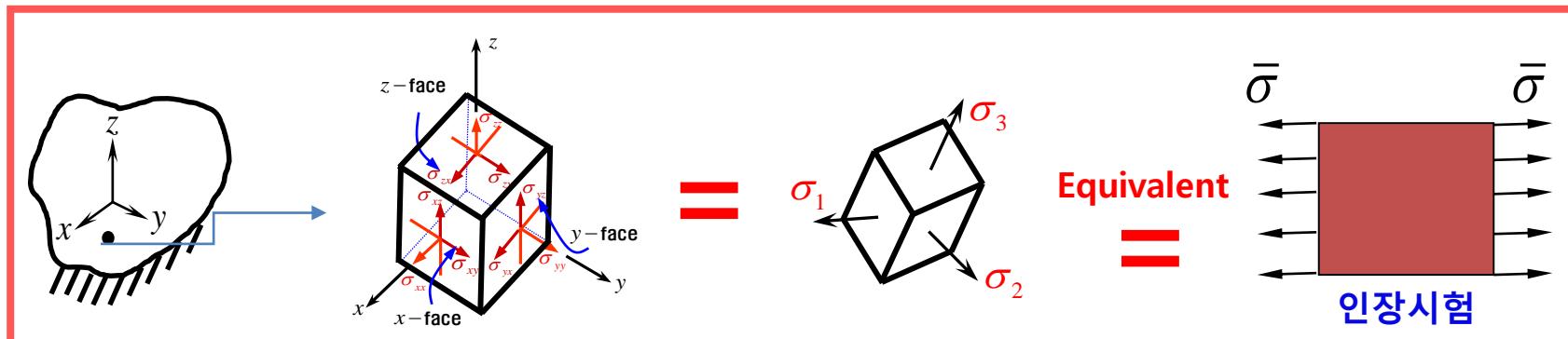
$$\sigma'_{ij} = \begin{pmatrix} \sigma_{xx} - \sigma_m & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} - \sigma_m & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} - \sigma_m \end{pmatrix}$$

편차응력텐서의 두번째 불변치 J_2 와 유효응력 $\bar{\sigma}$

$$J_2 = I_2 + \frac{I_1^2}{3} = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$\bar{\sigma} = \sqrt{3J_2} = \sqrt{\frac{3}{2} \sum \sum \sigma'_{ij} \sigma'_{ij}} = \sqrt{\frac{1}{2} [(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2)]}$$

$$= \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

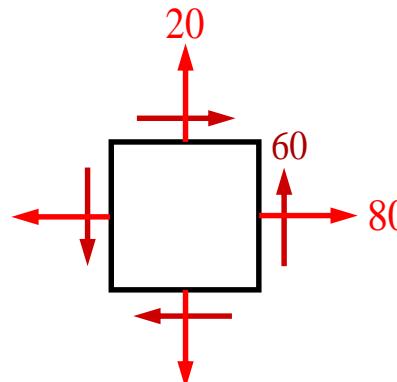


■ 주응력 및 주방향의 결정(고유치문제 방법) 예제

고유치문제와 특성방정식, 고유값(주응력)

$$\begin{bmatrix} 80 & 60 \\ 60 & 20 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \sigma \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \rightarrow \begin{vmatrix} 80 - \sigma & 60 \\ 60 & 20 - \sigma \end{vmatrix} = 0$$

$$(80 - \sigma)(20 - \sigma) - 3600 = 0 \quad \sigma_1 = 117.1, \quad \sigma_2 = -17.1$$

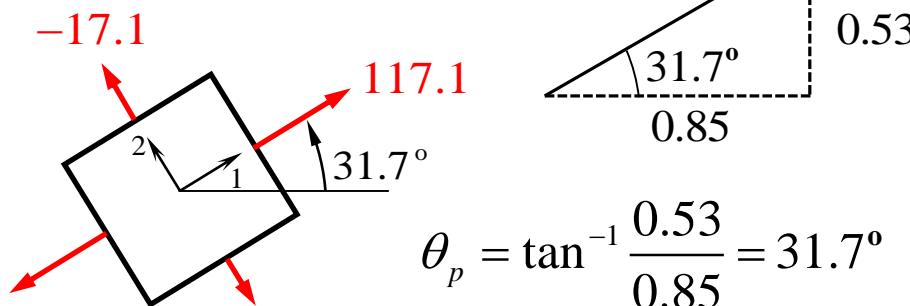


고유벡터(주응력축의 방향)

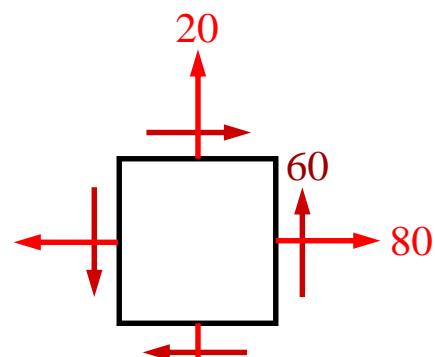
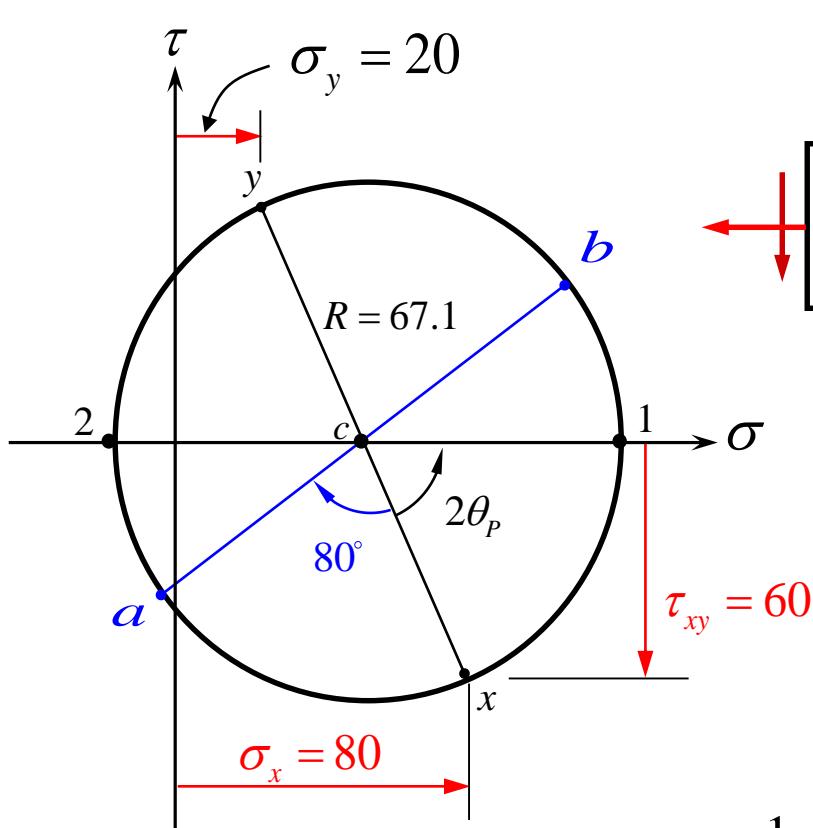
i) $\sigma_1 = 117.1 \Rightarrow \begin{bmatrix} 80 - 117.1 & 60 \\ 60 & 20 - 117.1 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\rightarrow \mathbf{n}^{(1)} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}^{(1)} = \begin{pmatrix} 0.85 \\ 0.53 \end{pmatrix}$

$$\Rightarrow -37.1n_1 + 60n_2 = 0$$

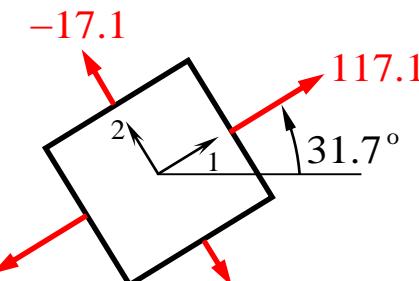
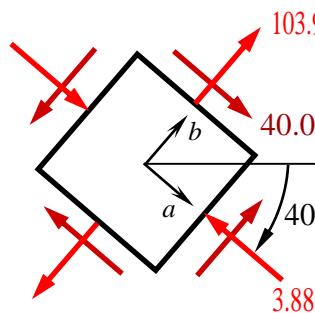
ii) $\sigma_1 = -17.1 \Rightarrow 60n_2 + 37.1n_1 = 0$



주응력 및 주방향의 결정(Mohr 원 방법) 예제

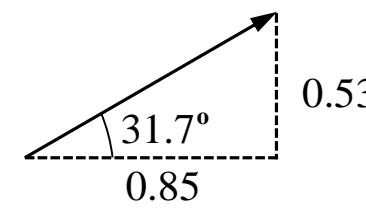


$$\begin{pmatrix} \sigma_{x'} & \tau_{x'y'} \\ \tau_{y'x'} & \sigma_{y'} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$



$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

$$\theta_p = \tan^{-1} \frac{0.53}{0.85} = 31.7^\circ$$

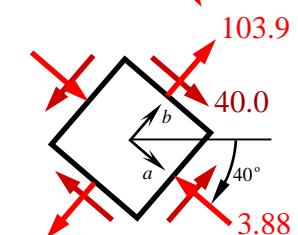
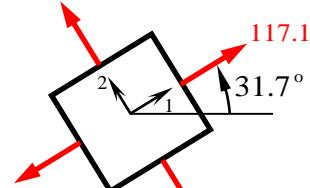
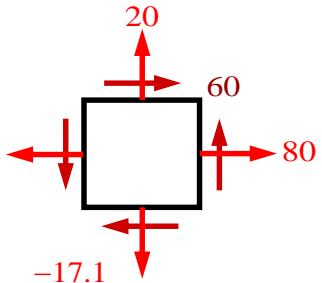


■ 주응력과 응력불변치

$$\square I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = \sigma_1 + \sigma_2 + \sigma_3$$

$$\begin{aligned}\square I_2 &= -\sigma_{xx}\sigma_{yy} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 \\ &= -\sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\end{aligned}$$

$$\begin{aligned}\square I_3 &= \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{xy}\sigma_{yz}\sigma_{zx} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2 \\ &= \sigma_1\sigma_2\sigma_3\end{aligned}$$



$$I_1 = 80 + 20 + 10 = 110$$

$$I_2 = -80 \times 20 - 20 \times 10 - 10 \times 80 + 60 \times 60 = 1000$$

$$I_3 = 80 \times 20 \times 10 + 2 \times 80 \times 0 - 80 \times 0 - 20 \times 0 - 10 \times 60 \times 60 = -20000$$

$$I_1 = 117.1 - 17.1 + 10 = 110$$

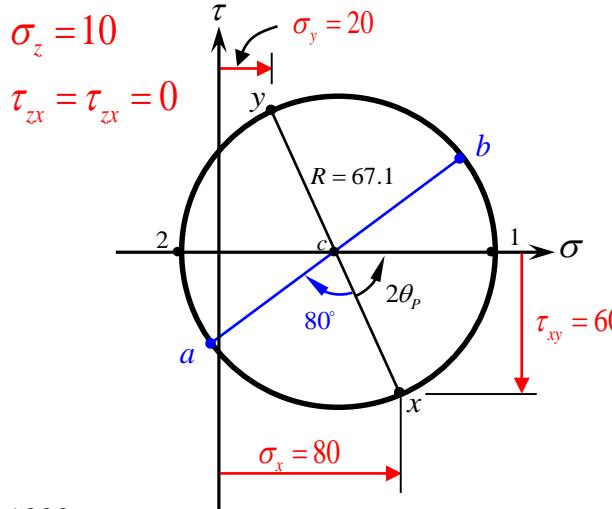
$$I_2 = -117.1 \times (-17.1) - (-17.1) \times 10 - 10 \times 117.1 = 1000$$

$$I_3 = 117.1 \times (-17.1) \times 10 = -20000$$

$$I_1 = 103.9 - 3.9 + 10 = 110$$

$$I_2 = -103.9 \times (-3.9) - (-3.9) \times 10 - 10 \times 103.9 + 40 \times 40 = 1000$$

$$I_3 = 103.9 \times (-3.9) \times 10 + 2 \times 40 \times 0 - 103.9 \times 0 - (-3.9) \times 0 - 10 \times 40 \times 40 = -20000$$



3.

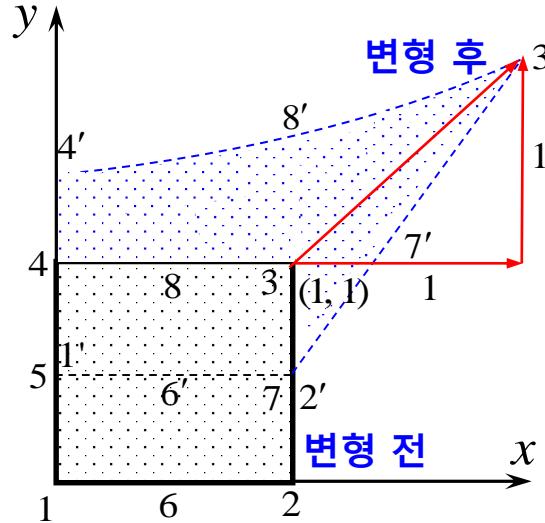
변형률과 변형률속도

변위장 및 속도장

변위장의 예

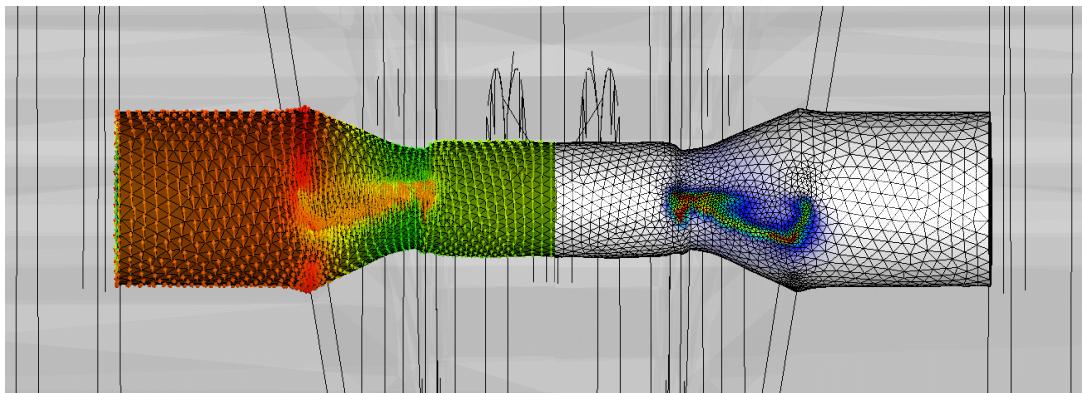
$$u_x(x, y) = xy$$

$$u_y(x, y) = \frac{1}{2}(x^2 y + 1)$$



변형 = 변위 - 강체운동

속도장의 예

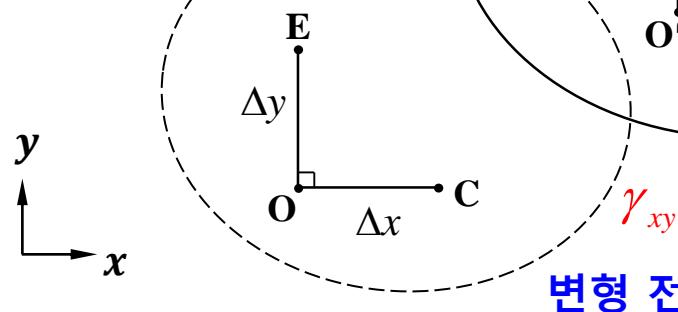


변형률의 정의

평면변형

$$u_x, u_y = f(x, y), u_z = 0$$

예 : 댐, 박판압연



평면변형의 변형률 텐서

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$u_x, u_y = f(x, y), u_z = 0$$

3차원에서 변형률 텐서

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

$$\varepsilon_{xy} = \varepsilon_{yx}, \quad \varepsilon_{yz} = \varepsilon_{zy}, \quad \varepsilon_{zx} = \varepsilon_{xz}$$

$$\cdot \varepsilon_{xx} = \lim_{c \rightarrow 0} \frac{\overline{O'C'} - \overline{OC}}{\overline{OC}}$$

$$\cdot \varepsilon_{yy} = \lim_{c \rightarrow 0} \frac{\overline{O'E'} - \overline{OE}}{\overline{OE}}$$

$$\cdot \gamma_{xy} = 2\varepsilon_{zy} = \frac{\pi}{2} = -\angle E'O'C'$$

변형률-변위의 관계

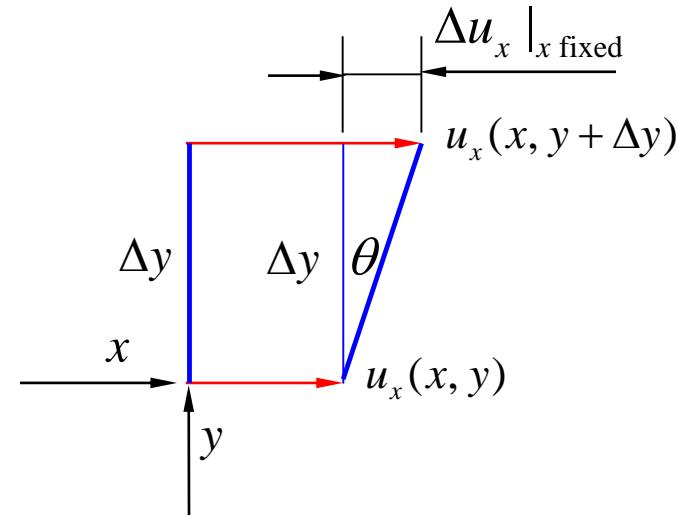
$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \quad \varepsilon_{xy} = \frac{1}{2} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y} \quad \varepsilon_{yz} = \frac{1}{2} \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right)$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} \quad \varepsilon_{zx} = \frac{1}{2} \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right)$$

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\frac{\partial u_x}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{\Delta u_x}{\Delta y} \Big|_{x \text{ fixed}}$$



$$\tan \theta = \frac{\Delta u_x |_{x \text{ fixed}}}{\Delta y}$$

$$\tan \theta = \theta, \theta \ll 1$$

변형률속도, 변형률속도-속도 관계식

변형률속도 $\dot{\varepsilon}_{ij}$

$$\varepsilon_{ij} \neq \int_0^t \dot{\varepsilon}_{ij} dt, \quad \dot{\varepsilon}_{ij} \neq \frac{d\varepsilon_{ij}}{dt}, \quad \Delta\varepsilon_{ij} = \dot{\varepsilon}_{ij}\Delta t$$

$$\dot{\varepsilon}_{ij} = \begin{pmatrix} \dot{\varepsilon}_{xx} & \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{xz} \\ \dot{\varepsilon}_{yx} & \dot{\varepsilon}_{yy} & \dot{\varepsilon}_{yz} \\ \dot{\varepsilon}_{zx} & \dot{\varepsilon}_{zy} & \dot{\varepsilon}_{zz} \end{pmatrix}$$

예제

$$t=10.000 \text{ (s)} \quad 100.0 \text{ mm}$$

$$t=10.001 \text{ (s)} \quad 101.0 \text{ mm}$$

변형률속도-속도 관계식

$$\dot{\varepsilon}_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

$$\dot{\varepsilon}_{xx} = \frac{\partial v_x}{\partial x}, \quad \dot{\varepsilon}_{yy} = \frac{\partial v_y}{\partial y}, \quad \dot{\varepsilon}_{zz} = \frac{\partial v_z}{\partial z}$$

$$\dot{\varepsilon}_{xy} = \frac{1}{2} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right), \quad \dot{\varepsilon}_{yz} = \frac{1}{2} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right), \quad \dot{\varepsilon}_{zx} = \frac{1}{2} \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right)$$

$$\Delta\varepsilon_{xx} = \frac{1.0}{100.0} = 0.01$$

$$\dot{\varepsilon}_{xx} = \frac{\Delta\varepsilon_{xx}}{\Delta t} = \frac{0.01}{0.001} = 10.0 \left(\frac{1}{s} \right)$$

$$\Delta\varepsilon_{xx} = \dot{\varepsilon}_{xx} \Delta t$$

■ 주변형률 및 유효변형률

변형률 관련 고유치 문제

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \varepsilon \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

고유치 - 주변형률

$$\varepsilon^3 - L_1\varepsilon^2 + L_2\varepsilon - L_3 = 0$$
$$\varepsilon = \varepsilon_1, \varepsilon_2, \varepsilon_3$$

주변형률

고유벡터 - 주변형률축

$$\varepsilon = \varepsilon_k \rightarrow n_i = n_i^{(k)}$$
$$\sum_i n_i^{(k)} n_i^{(l)} = 0 \quad (i \neq j)$$

주변형률 축은 직교함

평균변형률

$$\varepsilon_v = \varepsilon_m = \frac{L_1}{3} = \frac{\sum \varepsilon_{ii}}{3}$$

부피변화율

편차변형률 ε'_{ij}

$$\varepsilon'_{ij} = \begin{pmatrix} \varepsilon_{xx} - \varepsilon_m & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} - \varepsilon_m & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} - \varepsilon_m \end{pmatrix}$$

유효변형률

$$\bar{\varepsilon} = \sqrt{\frac{2}{3} \sum \sum \varepsilon'_{ij} \varepsilon'_{ij}} = \frac{\sqrt{2}}{3} \left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 \right]^{\frac{1}{2}}$$

주변형률속도 및 유효변형률속도/유효변형률

변형률속도 관련 고유치 문제

$$\begin{bmatrix} \dot{\varepsilon}_{xx} & \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{xz} \\ \dot{\varepsilon}_{yx} & \dot{\varepsilon}_{yy} & \dot{\varepsilon}_{yz} \\ \dot{\varepsilon}_{zx} & \dot{\varepsilon}_{zy} & \dot{\varepsilon}_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \dot{\varepsilon} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

고유치 - 주변형률속도

$$\begin{aligned} |\dot{\varepsilon}_{ij} - \dot{\varepsilon}\delta_{ij}| &= 0 \\ \dot{\varepsilon}^3 - \dot{L}_1\dot{\varepsilon}^2 + \dot{L}_2\dot{\varepsilon} - \dot{L}_3 &= 0 \\ \dot{\varepsilon} = \dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3 &\leftarrow \text{주변형률속도} \end{aligned}$$

고유벡터 - 주변형률속도축

$$\begin{aligned} \dot{\varepsilon} = \dot{\varepsilon}_k &\rightarrow n_i = n_i^{(k)} \\ n_i^{(k)}n_i^{(l)} &= 0 \quad (i \neq j) \end{aligned}$$

주변형률속도 축은 직교함

평균변형률속도

$$\dot{\varepsilon}_v = \dot{\varepsilon}_m = \frac{\dot{L}_1}{3} = \frac{\sum \dot{\varepsilon}_{ii}}{3}$$

부피변화율속도

편차변형률속도 $\dot{\varepsilon}'_{ij}$

$$\dot{\varepsilon}'_{ij} = \begin{pmatrix} \dot{\varepsilon}_{xx} - \dot{\varepsilon}_m & \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{xz} \\ \dot{\varepsilon}_{yx} & \dot{\varepsilon}_{yy} - \dot{\varepsilon}_m & \dot{\varepsilon}_{yz} \\ \dot{\varepsilon}_{zx} & \dot{\varepsilon}_{zy} & \dot{\varepsilon}_{zz} - \dot{\varepsilon}_m \end{pmatrix}$$

유효변형률

$$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3} \sum \sum \dot{\varepsilon}'_{ij} \dot{\varepsilon}'_{ij}} = \frac{\sqrt{2}}{3} \left[(\dot{\varepsilon}_1 - \dot{\varepsilon}_2)^2 + (\dot{\varepsilon}_2 - \dot{\varepsilon}_3)^2 + (\dot{\varepsilon}_3 - \dot{\varepsilon}_1)^2 \right]^{\frac{1}{2}}$$

유효변형률($\bar{\varepsilon}$)과 유효변형률속도($\dot{\bar{\varepsilon}}$)와의 관계

$$\bar{\varepsilon} = \int_0^1 \dot{\bar{\varepsilon}} dt \quad \left(\bar{\varepsilon}_{ij} \neq \int_0^1 \dot{\varepsilon}_{ij} dt \right)$$

4.

항복이론과 소성변형

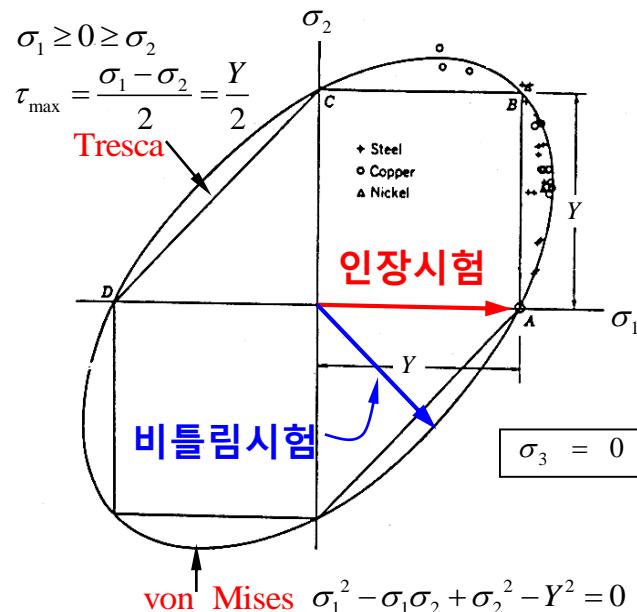
Huber-von Mises 항복이론

$$f = J_2 - k^2 = \frac{1}{2} \sum \sum \sigma'_{ij} \sigma'_{ij} - k^2 = 0, \quad \left(k = \frac{Y}{\sqrt{3}} \right)$$

$$\bar{\sigma} = \sqrt{\frac{1}{2} \left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right\}} = Y$$

$$\bar{\sigma} = \sqrt{\frac{1}{2} \left\{ (\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2) \right\}} = Y$$

평면응력에서 항복궤적



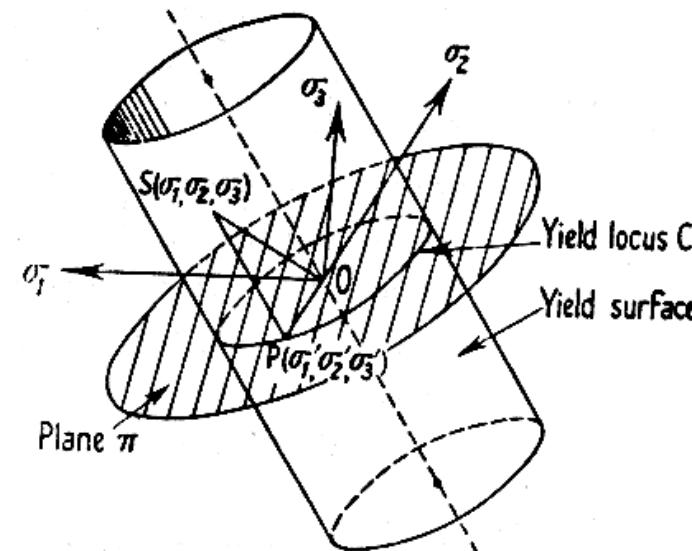
Tresca 항복이론

$$f = \tau_{\max} - k = 0$$

$$k = \frac{Y}{2}$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

3차원 응력공간에서 항복곡면



소성유동법칙

“비압축성 재료의 항복 시 변형률속도 텐서는 항복곡면에 직교함(Drucker의 가설의 결과)”

$$\dot{\varepsilon}_{ij} = \dot{\lambda} \frac{\partial f}{\partial \sigma'_{ij}} \quad \text{직교성, } f=0, \dot{\varepsilon}_{ij} \propto \nabla f$$

$$\Delta \varepsilon_{ij} = \Delta \lambda \frac{\partial f}{\partial \sigma'_{ij}}$$

$$\frac{\dot{\varepsilon}_{xx}}{\sigma'_{xx}} = \frac{\dot{\varepsilon}_{yy}}{\sigma'_{yy}} = \frac{\dot{\varepsilon}_{zz}}{\sigma'_{zz}} = \frac{\dot{\varepsilon}_{xy}}{\sigma'_{xy}} = \frac{\dot{\varepsilon}_{yz}}{\sigma'_{yz}} = \frac{\dot{\varepsilon}_{zx}}{\sigma'_{zx}} = \dot{\lambda}$$

von Mises 항복이론을 따를 경우

$$f(\sigma'_{pq}) = \frac{1}{2} \sigma'_{ij} \sigma'_{ij} - k^2 \quad \nabla f = \frac{\partial f}{\partial \sigma'_{ij}} = \frac{\partial}{\partial \sigma'_{ij}} \left(\frac{1}{2} \sigma'_{pq} \sigma'_{pq} - k^2 \right) = \frac{1}{2} (\delta_{ip} \delta_{jq} \sigma'_{pq} + \sigma'_{pq} \delta_{ip} \delta_{iq}) = \frac{1}{2} (\sigma'_{ij} + \sigma'_{ji}) = \sigma'_{ij}$$

$$\dot{\varepsilon}_{ij} = \dot{\lambda} \frac{\partial f}{\partial \sigma'_{ij}} = \dot{\lambda} \sigma'_{ij}, \quad \Delta \varepsilon_{ij} = \Delta \lambda \frac{\partial f}{\partial \sigma'_{ij}} \rightarrow \sigma'_{ij} = \frac{1}{\dot{\lambda}} \dot{\varepsilon}_{ij}$$

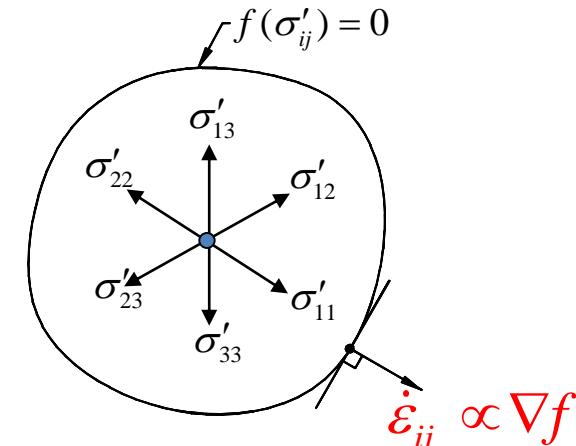
$$\dot{\lambda} = \frac{3}{2} \frac{\dot{\varepsilon}}{\bar{\sigma}} \quad \leftarrow \dot{\varepsilon} = \sqrt{\frac{2}{3} \dot{\varepsilon}'_{ij} \dot{\varepsilon}'_{ij}}, \quad \bar{\sigma} = \sqrt{3 J_2} = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}}$$

$$\sigma'_{ij} = \frac{2}{3} \frac{\bar{\sigma}}{\dot{\varepsilon}} \dot{\varepsilon}_{ij} = \frac{2}{3} \frac{Y}{\sqrt{\frac{2}{3} \dot{\varepsilon}'_{kl} \dot{\varepsilon}'_{kl}}} \dot{\varepsilon}_{ij}$$

사용자 입력 : 유동응력
Flow stress
탄성영역에서 0임.
수치 문제 발생시킴

덧셈부호규약 사용

항복곡면(Yield surface)



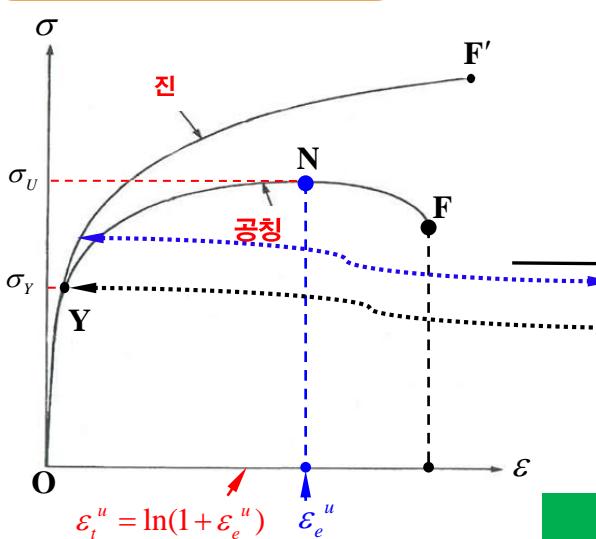
변형의 항복곡면 직교성

▣ 변형경화

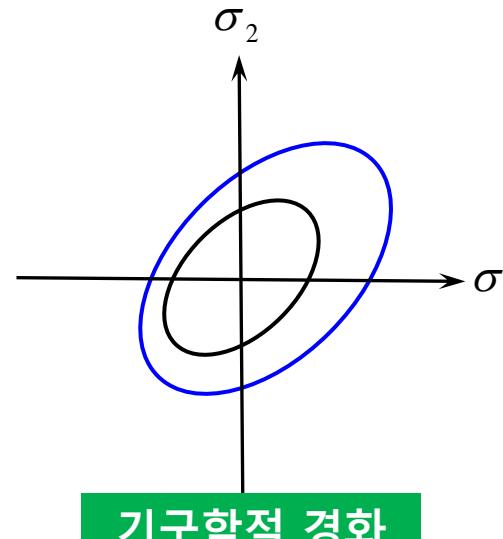
변형경화 발생 원인

- 소성변형은 전위(dislocation)에 의하여 발생
- 전위는 원자간의 슬립 또는 트위스트에 의하여 발생하는 일종의 결함적 요소
- 기 발생한 전위는 새로운 전위의 발생을 저지함 = **변형경화**

변형경화 모델



등방성 경화



기구학적 경화

변형과 유동응력의 이상화

변형의 이상화

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^p + \dot{\varepsilon}_{ij}^d$$

· $\dot{\varepsilon}_{ij}^p$: 소성변형률속도(Plastic strain rate)

· $\dot{\varepsilon}_{ij}^d$: 차이변형률속도(Difference strain rate)

탄소성(Elastoplastic) $\dot{\varepsilon}_{ij}^d = \dot{\varepsilon}_{ij}^e$

· $\dot{\varepsilon}_{ij}^e$: 탄성변형률속도(Elastic strain rate)

강소성(Rigid-plastic) $\dot{\varepsilon}_{ij}^d = 0$

상온에서의 주요 유동응력 모델

$$\bar{\sigma} = Y_o \left(1 + \frac{\bar{\varepsilon}}{b} \right)^n, \quad \bar{\sigma} = K \bar{\varepsilon}^n, \quad \bar{\sigma} = Y_o + K \bar{\varepsilon}^n$$

· Y_o : 초기항복응력(Initial yield stress)

· K : 강도계수(Strength coefficient)

· n : 변형경화지수(Strain hardening exponent)

고온에서의 주요 유동응력 모델

$$\bar{\sigma} = C \bar{\varepsilon}^n \dot{\varepsilon}^m, \quad \bar{\sigma} = C \dot{\varepsilon}^m$$

· n : 변형경화지수

· m : 변형률속도 의존도

· C : 고온강도계수

유동응력의 이상화

완전소성 : $\bar{\sigma} = \text{일정}$

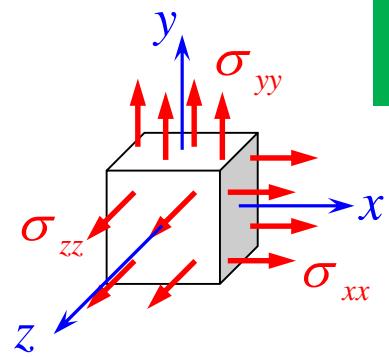
탄소성 : $\bar{\sigma} = Y(\bar{\varepsilon}^e, \bar{\varepsilon}^p)$

강소성 : $\bar{\sigma} = Y(\bar{\varepsilon}^p)$

강점소성 : $\bar{\sigma} = Y(\bar{\varepsilon}^p, \dot{\varepsilon}^p)$

강열점소성 : $\bar{\sigma} = Y(\bar{\varepsilon}^p, \dot{\varepsilon}^p, T)$

■ 등방성 재료에 대한 일반화된 후크법칙



가정 : 작은 변형, 등방성
중첩원리 적용 가능

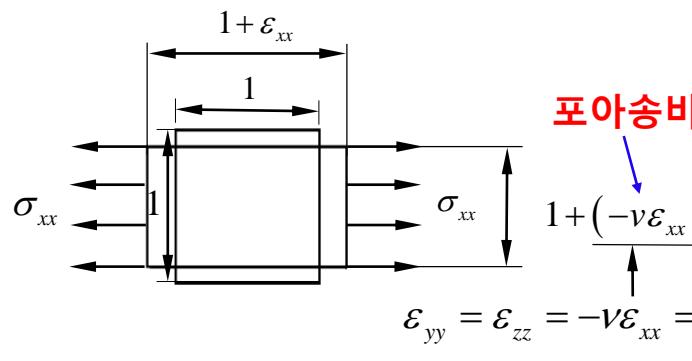
$$\tau_{ij} = G\gamma_{ij}$$

$$\varepsilon_{ii} = 0$$

$$G = \frac{E}{2(1+\nu)}$$

전단탄성계수

포아송비



$$\begin{aligned}
 \varepsilon_{xx} &= \frac{\sigma_{xx}}{E} & \varepsilon_{yy} &= \frac{\sigma_{yy}}{E} & \varepsilon_{zz} &= \frac{\sigma_{zz}}{E} \\
 \varepsilon_{yy} &= \varepsilon_{zz} = -\nu\varepsilon_{xx} & \varepsilon_{xx} &= \varepsilon_{zz} = -\nu\varepsilon_{yy} & \varepsilon_{xx} &= \varepsilon_{yy} = -\nu\varepsilon_{zz} \\
 \gamma_{xy} &= \gamma_{yz} = \gamma_{zx} = 0 & \gamma_{xy} &= \gamma_{yz} = \gamma_{zx} = 0 & \gamma_{xy} &= \gamma_{yz} = \gamma_{zx} = 0
 \end{aligned}$$

$$\left\{
 \begin{aligned}
 \varepsilon_{xx} &= \frac{1}{E} [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})] \\
 \varepsilon_{yy} &= \frac{1}{E} [\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})] \\
 \varepsilon_{zz} &= \frac{1}{E} [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})] \\
 \gamma_{xy} &= \frac{1}{G} \tau_{xy}, \quad \gamma_{yx} = \frac{1}{G} \tau_{yx}, \quad \gamma_{zx} = \frac{1}{G} \tau_{zx}
 \end{aligned}
 \right.$$

등방성
재료에
대한
일반화된
후크법칙

열팽창 고려 등방성 재료에 대한 일반화된 후크법칙

열팽창

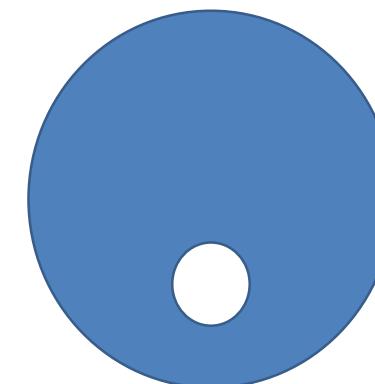
$$\delta = \alpha \Delta T L$$

단순 길이를 의미함

$$\varepsilon_T = \delta / L = \alpha \Delta T$$



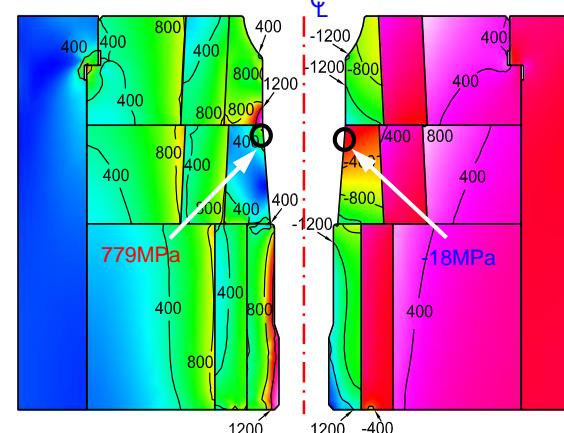
문제



$$\left\{ \begin{array}{l} \varepsilon_{xx} = \frac{1}{E} [\sigma_{xx} - v(\sigma_{yy} + \sigma_{zz})] + \alpha \square T \\ \varepsilon_{yy} = \frac{1}{E} [\sigma_{yy} - v(\sigma_{xx} + \sigma_{zz})] + \alpha \square T \\ \varepsilon_{zz} = \frac{1}{E} [\sigma_{zz} - v(\sigma_{xx} + \sigma_{yy})] + \alpha \square T \\ \gamma_{xy} = \frac{1}{G} \tau_{xy}, \gamma_{yx} = \frac{1}{G} \tau_{yx}, \gamma_{zx} = \frac{1}{G} \tau_{zx} \end{array} \right.$$

열팽창계수

등방성
재료에
대한
일반화된
후크법칙

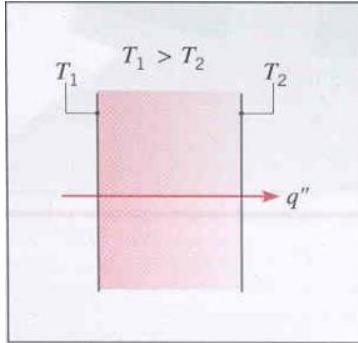


5.
리

열전도방정식 및 마찰, 역학 총정

고체에서 열전달 현상

열전도법칙

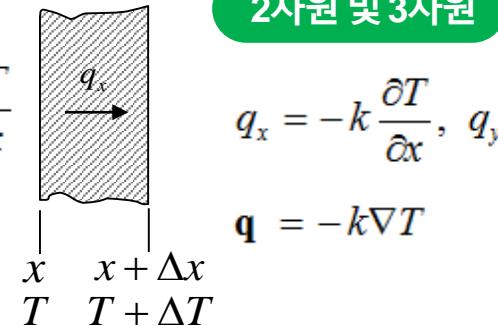


1차원

$$q_x = -k \frac{\Delta T}{\Delta x}, \quad -k \frac{dT}{dx}$$

열전도계수

Fourier의 열전도법칙

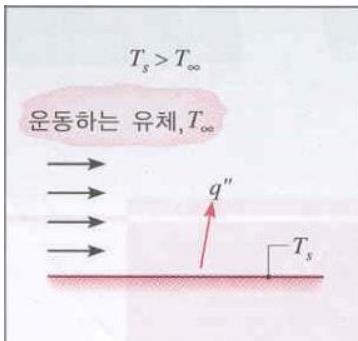


2차원 및 3차원

$$q_x = -k \frac{\partial T}{\partial x}, \quad q_y = -k \frac{\partial T}{\partial y}, \quad q_z = -k \frac{\partial T}{\partial z}$$

$$\mathbf{q} = -k \nabla T$$

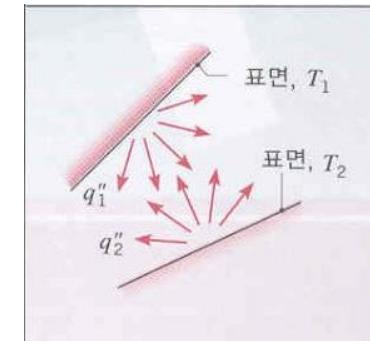
열대류법칙



$$q_c = h_q (T - T_\infty)$$

열전달계수

열방사법칙

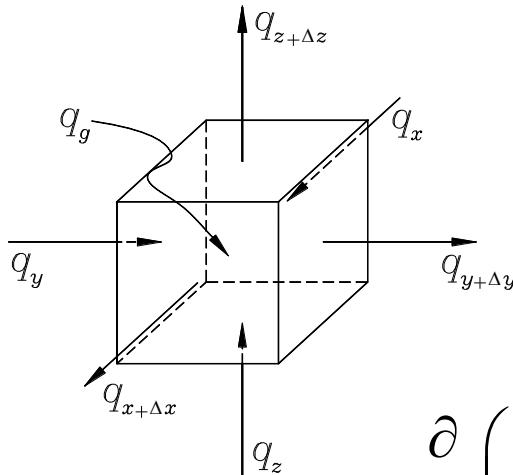


$$q_r = \sigma \epsilon (T^4 - \bar{T}_q^4)$$

Stefan-Boltzmann 상수

▣ 고체의 열전달 - 경계치문제

열전도방정식



경계조건

$$T = \bar{T} \quad \text{on } \mathbf{S}_T$$

$$k \nabla T \cdot \mathbf{n} = k T_{,i} n_i = -\sigma \varepsilon (T^4 - \bar{T}_q^4) - h_q (T - \bar{T}_q) \quad \text{on } \mathbf{S}_q$$

덧셈 부호규약 사용:
콤마(,)뒤의 첨자는 편미분을 의미함

$$T_{,i} n_i = \sum_i \frac{\partial T}{\partial x_i} n_i$$

$$(-q_{x+\Delta x} + q_x) \Delta y \Delta z + (-q_{y+\Delta y} + q_y) \Delta z \Delta x$$

$$+ (-q_{z+\Delta z} + q_z) \Delta x \Delta y + q_g \Delta x \Delta y \Delta z = \rho c \frac{\partial T}{\partial t} \Delta x \Delta y \Delta z$$

$$q_{x+\Delta x} = q_x + \frac{\partial q_x}{\partial x} \Delta x, \quad etc.$$

$$-q_{x+\Delta x} + q_x = -\frac{\partial q_x}{\partial x} \Delta x = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)$$

열용량

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q_g = \rho c \frac{\partial T}{\partial t}$$

$$q_g = (0.9 \square 1.0) \bar{\sigma} \dot{\varepsilon}$$

열전도방정식

▣ 마찰법칙과 질량보존의 법칙

특수경계조건: 마찰

쿨롱(Coulomb) 마찰법칙 : $|\sigma_t| \leq \mu |\sigma_n|$

μ : **마찰계수** (Coefficient of Coulomb friction)

$$\sigma_t = -\mu \sigma_n g(v_t) \text{ on } S_C$$

하이브리드(Hybrid) 마찰법칙 :

$$\sigma_t = -\mu \sigma_n g(v_t) \text{ on } S_C \quad \text{when } \mu \sigma_n < m'k$$

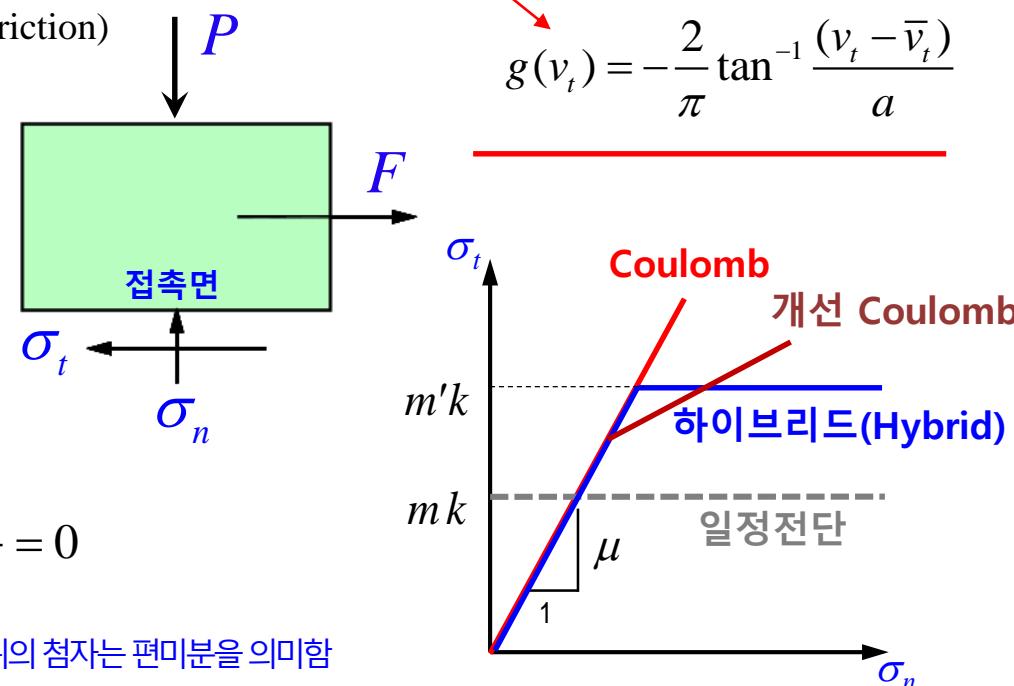
$$\sigma_t = m'k g(v_t) \text{ on } S_C \quad \text{when } \mu \sigma_n \geq m'k$$

일정전단마찰법칙 : $|\sigma_t| \leq mk$

m : **마찰상수** (Coefficient of Coulomb friction)

$$\sigma_t = mkg(v_t) \text{ on } S_C$$

등식은 미끄러질 때 성립함



질량보존법칙

비압축성 조건

$$\varepsilon_{ii} = u_{i,i} = 0$$

$$\dot{\varepsilon}_{ii} = v_{i,i} = 0$$

질량보존법칙

$$(\rho v_i)_{,i} + \frac{\partial \rho}{\partial t} = 0$$

덧셈 부호규약 사용: 콤마(,) 뒤의 첨자는 편미분을 의미함

■ 역학 총정리

구분	상세 법칙	방정식 및 관계식	비고
Newton 운동	평형조건식	$\sum F_i = m a_i, \sum M_i = I \alpha_i$	
	평형방정식	$\sigma_{ji,j} + f_i = 0, \sigma_{ij} = \sigma_{ji}$	Navier-Cauchy 방정식
	운동방정식	$\sigma_{ji,j} + f_i = \rho \dot{v}_i$	Navier-Stokes 방정식
에너지 보존	열전도방정식	$(k \phi_{,i})_{,i} + q_g = \rho c \frac{\partial \phi}{\partial t}$	고체에 적용
	에너지보존방정식	$(k \phi_{,i})_{,i} + q_g = \rho c \left(\frac{\partial \phi}{\partial t} + \phi_{,j} v_j \right)$	유체에 적용
질량 보존	연속방정식	$v_{i,i} = 0$	비압축성 재료
		$(\rho v_i)_{,i} + \frac{\partial \rho}{\partial t} = 0$	압축성 재료
구성 방정식	Hooke 법칙	$\sigma_{ij} = 2\mu \varepsilon_{ij} + \lambda \varepsilon_{kk} \delta_{ij}, \varepsilon_{ij} = \frac{1}{E} [(1+\nu) \sigma_{ij} - \nu \sigma_{kk} \delta_{ij}]$	열역학 2법칙이 성립하도록 함
	소성유동법칙	$\sigma'_{ij} = \frac{2}{3} \frac{\bar{\sigma}}{\bar{\varepsilon}} \dot{\varepsilon}_{ij}$	
	Fourier의 열전도법칙	$q_i = -k \phi_{,i}$	
		
변형의 기하학	변위-변형률 관계식	$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})$	
	속도-변형률속도 관계식	$\dot{\varepsilon}_{ij} = \frac{1}{2} (v_{i,j} + v_{j,i})$	
경계 조건	필수경계조건	$u_i = \bar{u}_i, v_i = \bar{v}_i, \phi = \bar{\phi}$	
	자연경계조건	$t_i^{(n)} = \sigma_{ji} n_j = \bar{t}_i, q_i = -k \phi_{,i} = \bar{q}_i$	